Robert Lazarsfeld. Pretalk: Positivity for Divisors.

X = smooth proj var/$\mathbb{C}$. dim n
D = Divisor on X

Recall. D = ample if $\varphi_{|mD|} : X \hookrightarrow \mathbb{P}^N$ ~~birational onto image~~
is embedding for $m >> 0$.

D = big if $\varphi_{|mD|} : X \dashrightarrow \mathbb{P}^N$ birational onto image.

$$\left(\Leftrightarrow h^0(mD) \sim m^{\dim X} \right)$$

Ask: How can we get sense of all divisors that are big or ample?

1960's - Kleiman et al. "numerical theory of positivity".

Notions make sense for $\mathbb{Q}$-divisors. Henceforth work with these.

(1) Numerical Equivalence. Def: Say $D_1 \sim D_2$ (num equiv) if
$(D_1 \cdot C) = (D_2 \cdot C)$ all curves.

$N^1(X)_{\mathbb{Z}}$, $N^1(X)_{\mathbb{Q}}$ numerical equivalence classes.

Prop: $N^1(X)_{\mathbb{Z}}$ f.g. free ab gp.
$N^1(X)_{\mathbb{Q}}$ f.dim $\mathbb{Q}$-v.s.

$$\begin{array}{l} H^2(X, \mathbb{Z}) \quad \text{f.g. ab gp} \\ \quad \cup \\ \left\{ \begin{array}{c} \text{classes of} \\ \text{divisors} \end{array} \right\} \longrightarrow N^1(X)_{\mathbb{Z}} \end{array}$$